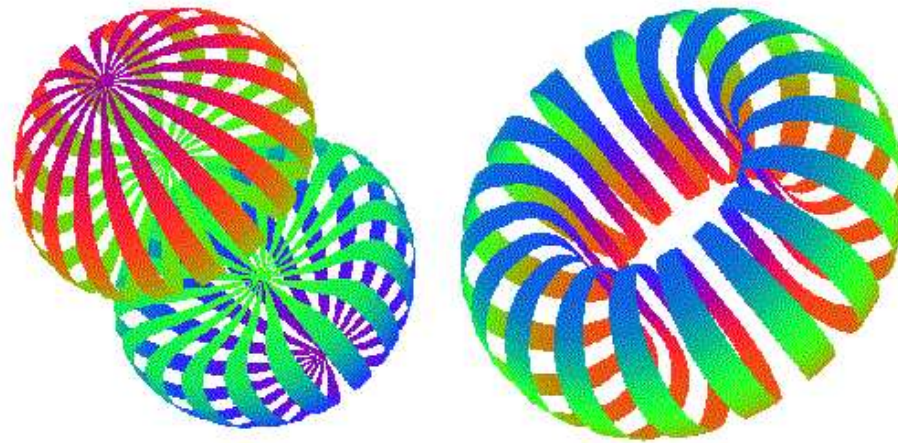


# Nuclear Forces and *Ab Initio* Calculations of Nuclei

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Machleidt, *Adv. Nucl. Phys.* **19**, 189 (1989)

Carlson & Schiavilla, *Rev. Mod. Phys.* **70**, 743 (1997)

Pieper & Wiringa, *Ann. Rev. Nucl. Part. Sci.* **51**, 53 (2002)

<http://unedf.org>

# *Ab Initio* CALCULATIONS OF NUCLEI

## GOALS

Understand nuclei at the level of elementary interactions between individual nucleons, including

- Binding energies, excitation spectra, relative stability
- Densities, electromagnetic moments, transition amplitudes, cluster-cluster overlaps
- Low-energy  $NA$  &  $AA$  scattering, asymptotic normalizations, astrophysical reactions

## REQUIREMENTS

- Two-nucleon potentials that accurately describe elastic  $NN$  scattering data
- Consistent three-nucleon potentials and electroweak current operators
- Precise methods for solving the many-nucleon Schrödinger equation

## METHODS

- Faddeev, hyperspherical harmonic, AGS, etc., for  $A = 3, 4$  ground states & scattering
- Variational & Green's function Monte Carlo calculations for  $A \leq 12$  nuclei
- Auxiliary field diffusion Monte Carlo for  $N \leq 40$  neutron drops
- No-core shell model for  $A \leq 16+$
- Coupled-cluster for  $A = 4, 16, 40$  closed-shell and near closed-shell

# PROGRESS IN *Ab Initio* CALCULATIONS

## Accurate Representations of Nuclear Forces

- Yukawa meson-exchange theory (1935)    Fujita-Miyazawa three-nucleon potential (1957)  
First phase-shift analysis of  $NN$  scattering data (1957)  
Gammel-Thaler, Hamada-Johnston, Reid phenomenological potentials (1957-1968)  
Bonn, Nijmegen, Paris field-theoretic models (1970s)  
Tucson-Melbourne, Brazil, Urbana  $NNN$  potentials (1970s-80s)  
Nijmegen partial wave analysis (PWA93)  $\rightarrow \chi^2 \sim 1$  (1993)  
Nijm I, Nijm II, Argonne  $v_{18}$ , CD-Bonn (1990s)     $\chi$  Effective field theory at  $N^3LO$  (2000s)

## Accurate Solutions of Many-Body Schrödinger Equation

- $^2H$  by Numerical Integration (1952)    "The computation takes between 5 and 20 minutes,  
 $^3H$  by Faddeev (1975-85)    the printing takes another 5 minutes"  
 $^4He$  by Green's function Monte Carlo (1988)  
 $A = 6$  by No-core shell model & GFMC (1994-95)  
 $A = 7$  by GFMC & NCSM (1997-98)  
 $A = 8$  by GFMC & NCSM (2000)  
 $^4He$  benchmark by 7 methods to 0.1% (2001)  
 $A = 9, 10$  by GFMC & NCSM (2002)  
 $^{12}C$  by GFMC & NCSM (2004-)  
 $^{16}O, ^{40}Ca$  by coupled-cluster (2005-7)

# THE NUCLEAR MANY-BODY PROBLEM

Many-Body Schrödinger Equation (MBSE) for bound states:

$$\begin{aligned} H\Psi(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_A; s_1, s_2, \dots, s_A; t_1, t_2, \dots, t_A) \\ = E\Psi(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_A; s_1, s_2, \dots, s_A; t_1, t_2, \dots, t_A) \end{aligned}$$

where

$\mathbf{r}_i$  are the nucleon coordinates in r-space

$s_i$  are the nucleon spins ( $= \pm \frac{1}{2}$ )

$t_i$  are the nucleon isospins ( $p$  or  $n = \pm \frac{1}{2}$ )

This corresponds to

$2^A \times \binom{A}{Z}$  coupled second-order differential equations in  $3A$  dimensions!

which is

96 for  ${}^4\text{He}$

17,920 for  ${}^8\text{Be}$

3,784,704 for  ${}^{12}\text{C}$

This is a challenging many-body problem!

# VARIATIONAL MONTE CARLO

Minimize expectation value of  $H$

$$E_V = \frac{\langle \Psi_V | H | \Psi_V \rangle}{\langle \Psi_V | \Psi_V \rangle} \geq E_0$$

Trial function (s-shell nuclei)

$$|\Psi_V\rangle = \left[ \mathcal{S} \prod_{i<j} (1 + U_{ij} + \sum_{k \neq i,j} U_{ijk}) \right] |\Psi_J\rangle$$

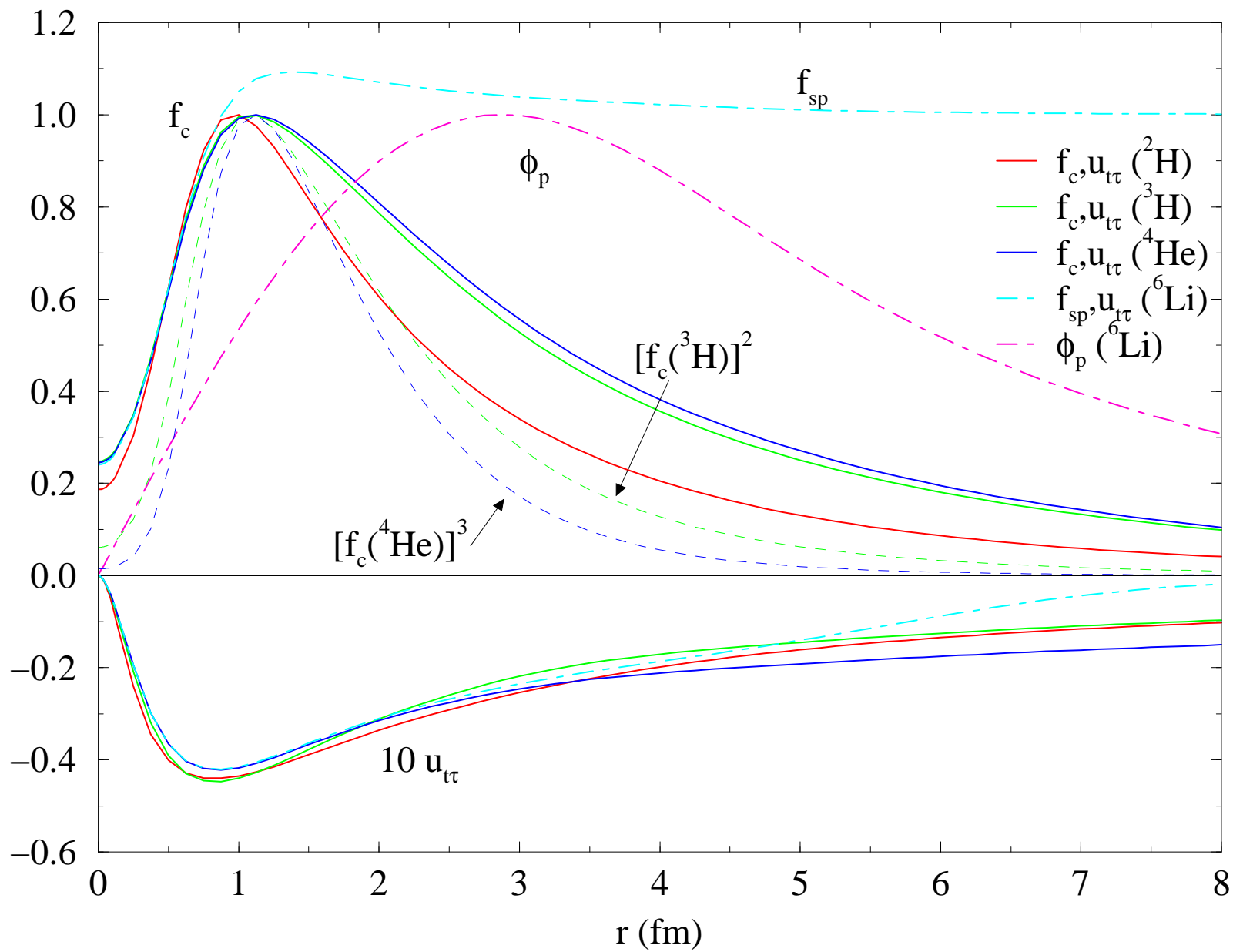
$$|\Psi_J\rangle = \left[ \prod_{i<j} f_c(r_{ij}) \right] |\Phi_A(JMTT_3)\rangle$$

$$|\Phi_d(1100)\rangle = \mathcal{A} | \uparrow p \uparrow n \rangle ; |\Phi_\alpha(0000)\rangle = \mathcal{A} | \uparrow p \downarrow p \uparrow n \downarrow n \rangle$$

$$U_{ij} = \sum_{p=2,6} u_p(r_{ij}) O_{ij}^p ; U_{ijk} = -\epsilon V_{ijk}(\tilde{r}_{ij}, \tilde{r}_{jk}, \tilde{r}_{ki})$$

Functions  $f_c(r_{ij})$  and  $u_p(r_{ij})$  obtained from coupled differential equations with  $v_{ij}$ .

## Correlation functions



## Trial function (p-shell nuclei)

$$|\Psi_J\rangle = \mathcal{A} \left\{ \prod_{i < j \leq 4} f_{ss}(r_{ij}) \sum_{LS[n]} \beta_{LS[n]} \prod_{k \leq 4 < l \leq A} f_{sp}(r_{kl}) \prod_{4 < l < m \leq A} f_{pp}(r_{lm}) \right. \\ \left. \left| \Phi_\alpha(0000)_{1234} \prod_{4 < l \leq A} \phi_p^{LS[n]}(\mathbf{R}_{\alpha l}) \{ [Y_1^{m_l}(\Omega_{\alpha l})]_{LM_L} \otimes [\chi_l(\frac{1}{2}m_s)]_{SM_S} \}_{JM} [\nu_l(\frac{1}{2}t_3)]_{TT_3} \right\rangle \right\}$$

## Diagonalization

in  $\beta_{LS[n]}$  basis to produce energy spectra  $E(J_x^\pi)$  and orthogonal excited states  $\Psi_V(J_x^\pi)$

## Expectation values

$\Psi_V(\mathbf{R})$  represented by vector with  $2^A \times \binom{A}{Z}$  spin-isospin components (or a little less assuming isospin conservation) for each space configuration  $\mathbf{R} = (\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_A)$ ; expectation values are evaluated in a Metropolis Monte Carlo random walk, i.e., by a summation over samples drawn from probability distribution  $W(\mathbf{R}) = |\Psi_P(\mathbf{R})|^2$ :

$$\frac{\langle \Psi_V | O | \Psi_V \rangle}{\langle \Psi_V | \Psi_V \rangle} = \sum \frac{\Psi_V^\dagger(\mathbf{R}) O \Psi_V(\mathbf{R})}{W(\mathbf{R})} / \sum \frac{\Psi_V^\dagger(\mathbf{R}) \Psi_V(\mathbf{R})}{W(\mathbf{R})}$$

$\Psi^\dagger \Psi$  is a dot product and  $\Psi^\dagger O \Psi$  a sparse matrix operation.

# A SIMPLIFIED VMC CALCULATION

- Generate a random position:  $\mathbf{R} = \mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_A$
- Make many (1000's) random steps based on the probability  $P = |\Psi_V(\mathbf{R})|^2$
- Start integration loop:
  - Make order 10 steps based on  $P$
  - Compute and sum  $H_{\text{local}}(\mathbf{R}) = [\Psi_V(\mathbf{R})^\dagger H \Psi_V(\mathbf{R})] / |\Psi_V(\mathbf{R})|^2$
- $\langle \Psi_V | H | \Psi_V \rangle / \langle \Psi_V | \Psi_V \rangle = \text{average}(H_{\text{local}})$

## Making a random step

- Use  $3A$  uniform random numbers on  $(0,1)$ ,  $\{w_j\}$ , to make  $\Delta\mathbf{R}$ ;  $\Delta x_i = 2\delta r(w_j - 1)$
- $\mathbf{R}' = \mathbf{R} + \Delta\mathbf{R}$ ;  $P(\Delta\mathbf{R}) = |\Psi_V(\mathbf{R}')|^2 / |\Psi_V(\mathbf{R})|^2$
- Make another random number on  $(0,1)$ :  $p$
- If  $P > p$ , the step is accepted; replace  $\mathbf{R}$  with  $\mathbf{R}'$   
if  $P < p$ , the step is rejected; discard  $\mathbf{R}'$  and stay at  $\mathbf{R}$

Gradients and Laplacians are computed by differences:  $6A$  evaluations of  $\Psi_V(\mathbf{R} + \delta_j \vec{r}_i)$

# SCALING OF VMC CALCULATION TIME WITH NUCLEUS

Scales with # particles (6A w.f. calculations for kinetic energy)  $\times$

# pairs (operations to construct w.f.)  $\times$  spin  $\times$  isospin (size of w.f. vector):

	$A$	Pairs	Spin $\times$ Isospin	$\Pi(/^8\text{Be})$
$^4\text{He}$	4	6	$16 \times 2$	0.001
$^5\text{He}$	5	10	$32 \times 5$	0.010
$^6\text{Li}$	6	15	$64 \times 5$	0.036
$^7\text{Li}$	7	21	$128 \times 14$	0.33
$^8\text{Be}$	8	28	$256 \times 14$	1.
$^9\text{Be}$	9	36	$512 \times 42$	8.7
$^{10}\text{Be}$	10	45	$1024 \times 90$	52.
$^{11}\text{B}$	11	55	$2048 \times 132$	200.
$^{12}\text{C}$	12	66	$4096 \times 132$	530.
$^{14}\text{C}$	14	91	$16384 \times 1001$	26,000.
$^{16}\text{O}$	16	120	$65536 \times 1430$	220,000.
$^{40}\text{Ca}$	40	780	$1.1 \times 10^{12} \times 6.6 \times 10^9$	$2.8 \times 10^{20}$
$^8\text{n}$	8	28	$256 \times 1$	0.071
$^{14}\text{n}$	14	91	$16384 \times 1$	26.

# GREEN'S FUNCTION MONTE CARLO

Projects out lowest energy state from variational trial function

$$\begin{aligned}\Psi(\tau) = \exp[-(H - E_0)\tau]\Psi_V &= \sum_n \exp[-(E_n - E_0)\tau]a_n\psi_n \\ \Psi(\tau \rightarrow \infty) &= a_0\psi_0\end{aligned}$$

Evaluation of  $\Psi(\tau)$  done stochastically in small time steps  $\Delta\tau$

$$\Psi(\mathbf{R}_n, \tau) = \int G(\mathbf{R}_n, \mathbf{R}_{n-1}) \cdots G(\mathbf{R}_1, \mathbf{R}_0) \Psi_V(\mathbf{R}_0) d\mathbf{R}_{n-1} \cdots d\mathbf{R}_0$$

using the short-time propagator accurate to order  $(\Delta\tau)^3$  ( $V_{ijk}$  term omitted for simplicity)

$$G_{\alpha\beta}(\mathbf{R}, \mathbf{R}') = e^{E_0\delta\tau} G_0(\mathbf{R}, \mathbf{R}') \langle \alpha | \left[ \mathcal{S} \prod_{i < j} \frac{g_{ij}(\mathbf{r}_{ij}, \mathbf{r}'_{ij})}{g_{0,ij}(\mathbf{r}_{ij}, \mathbf{r}'_{ij})} \right] | \beta \rangle$$

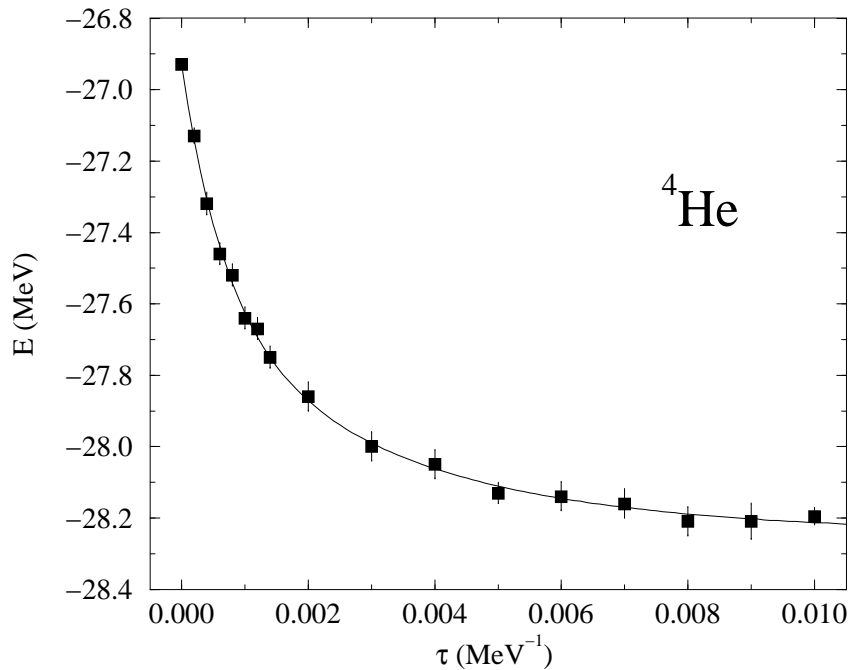
where the free many-body propagator is

$$G_0(\mathbf{R}, \mathbf{R}') = \langle \mathbf{R} | e^{-K\Delta\tau} | \mathbf{R}' \rangle = \left[ \sqrt{\frac{m}{2\pi\hbar^2\Delta\tau}} \right]^{3A} \exp \left[ \frac{-(\mathbf{R} - \mathbf{R}')^2}{2\hbar^2\Delta\tau/m} \right]$$

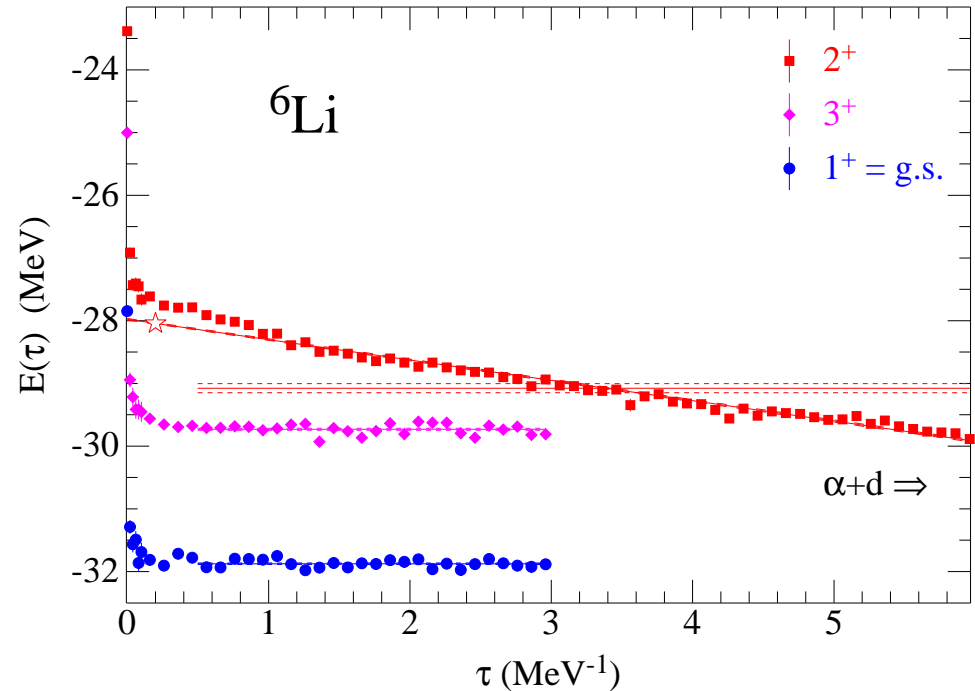
and  $g_{0,ij}$  and  $g_{ij}$  are the free and exact two-body propagators

$$g_{ij}(\mathbf{r}_{ij}, \mathbf{r}'_{ij}) = \langle \mathbf{r}_{ij} | e^{-H_{ij}\Delta\tau} | \mathbf{r}'_{ij} \rangle$$

# EXAMPLES OF GFMC PROPAGATION



- Curve has  $\exp(-E_i\tau)$  with  $E_i = 1480, 340$  &  $20.2$  MeV (20.2 MeV is first  ${}^4\text{He}$   $0^+$  excitation)
- $\Psi_V$  has small amounts of 1.5 GeV contamination



- g.s. ( $1^+$ ) &  $3^+$  stable after  $\tau = 0.2$   $\text{MeV}^{-1}$
- $2^+$  (a broad resonance) never stable – decaying to separated  $\alpha$  &  $d$
- $E(\tau=0.2)$  is best GFMC estimate of resonance energy

## Mixed estimates

$$\langle O(\tau) \rangle = \frac{\langle \Psi(\tau) | O | \Psi(\tau) \rangle}{\langle \Psi(\tau) | \Psi(\tau) \rangle} \approx \langle O(\tau) \rangle_{\text{Mixed}} + [\langle O(\tau) \rangle_{\text{Mixed}} - \langle O \rangle_V]$$
$$\langle O(\tau) \rangle_{\text{Mixed}} = \frac{\langle \Psi_V | O | \Psi(\tau) \rangle}{\langle \Psi_V | \Psi(\tau) \rangle} \quad ; \quad \langle H(\tau) \rangle_{\text{Mixed}} = \frac{\langle \Psi(\tau/2) | H | \Psi(\tau/2) \rangle}{\langle \Psi(\tau/2) | \Psi(\tau/2) \rangle} \geq E_0$$

Propagator cannot contain  $p^2$ ,  $L^2$ , or  $(\mathbf{L} \cdot \mathbf{S})^2$  operators:

$G_{\beta\alpha}(\mathbf{R}', \mathbf{R})$  has only  $v'_8$

$\langle v_{18} - v'_8 \rangle$  computed perturbatively with extrapolation (small for AV18)

Fermion sign problem limits maximum  $\tau$ :

$G_{\beta\alpha}(\mathbf{R}', \mathbf{R})$  brings in lower-energy boson solution

$\langle \Psi_V | H | \Psi(\tau) \rangle$  projects back fermion solution. but statistical errors grow exponentially

**Constrained-path propagation**, removes steps that have

$$\overline{\Psi^\dagger(\tau, \mathbf{R}) \Psi(\mathbf{R})} = 0$$

Possible systematic errors reduced by 10 – 20 unconstrained steps before evaluating observables.

## A SIMPLIFIED GFMC CALCULATION

- Start with collection of  $\Psi(\tau=0, \mathbf{R}_j) = \Psi_V(\mathbf{R}_j)$  from VMC calculation

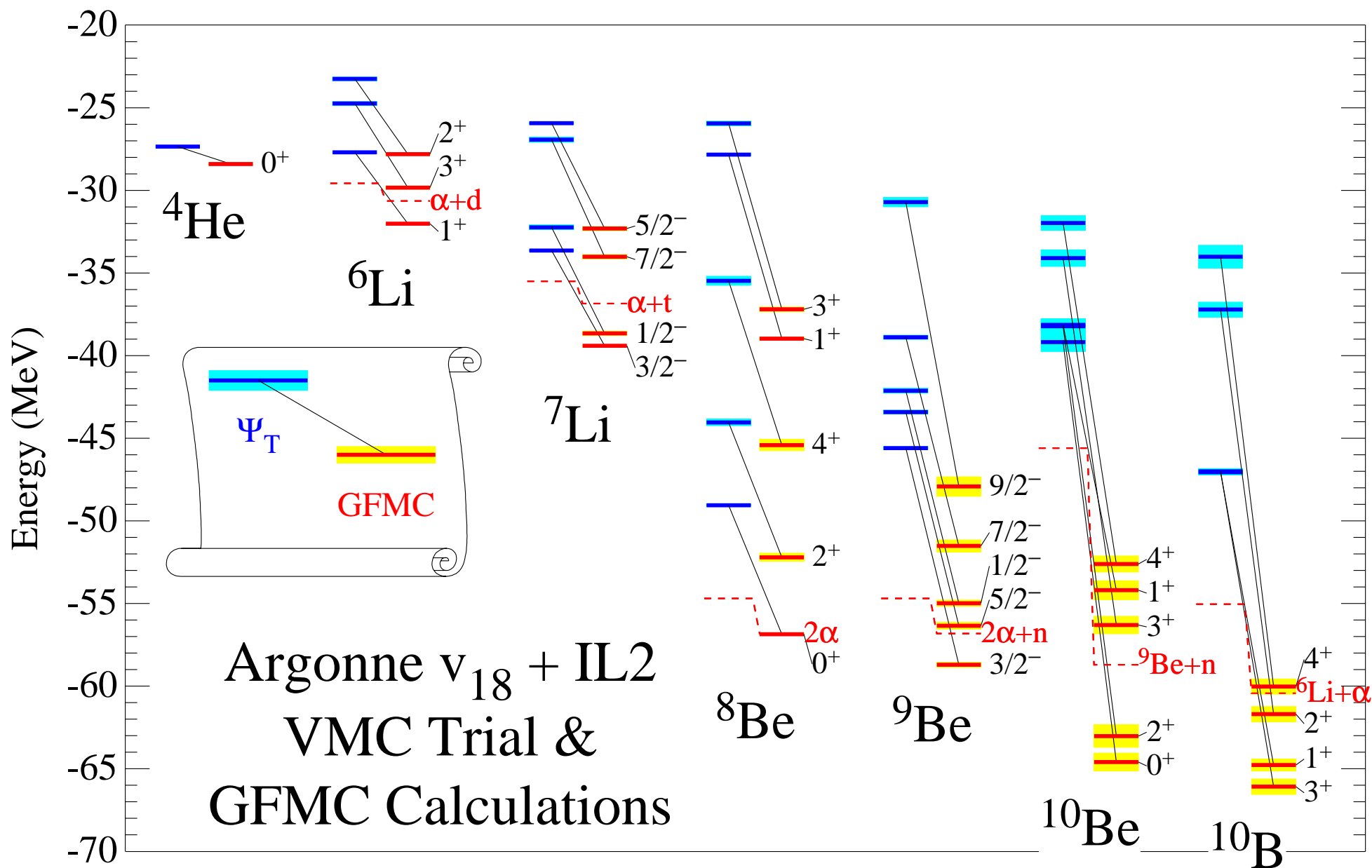
- Loop over time steps  $\tau_n = n\Delta\tau$

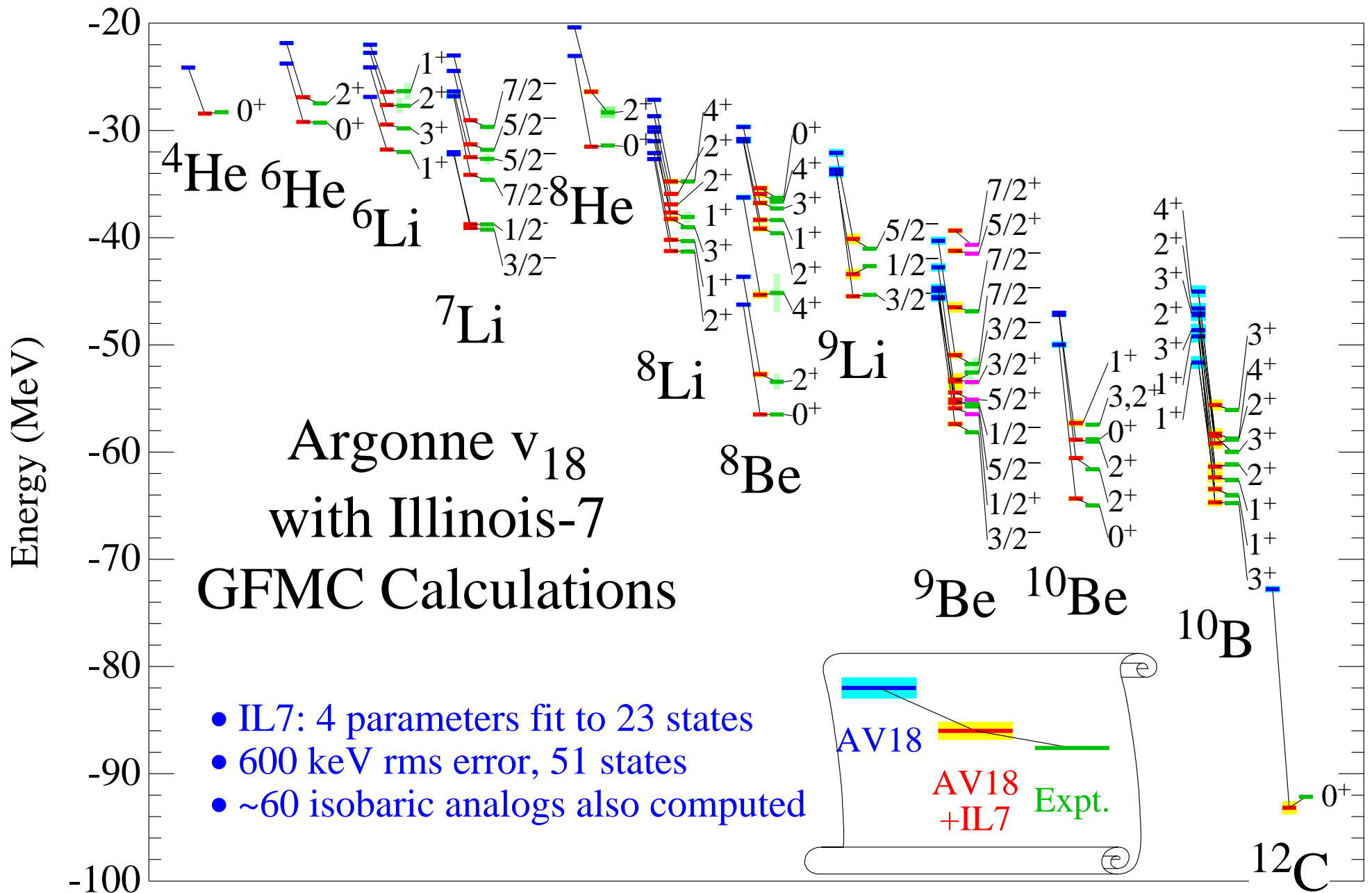
- Loop over configurations  $J$

- \* Make random step  $\mathbf{R}'_j = \mathbf{R}_j + \Delta\mathbf{R}_j$  by sampling the Gaussian  $\exp\left[\frac{-(\mathbf{R}'_j - \mathbf{R}_j)^2}{2\hbar^2 \Delta\tau/m}\right]$
- \* Sample several directions based on simplified  $\Psi_V$  and potential
- \* Compute  $\Psi(\tau_{n+1}, \mathbf{R}'_j) = G(\mathbf{R}'_j, \mathbf{R}_j)\Psi(\tau_n, \mathbf{R}_j)$
- \* Possibly mark as killed due to constraint  $\Psi^\dagger(\tau_{n+1}, \mathbf{R}'_j) \cdot \Psi_V(\mathbf{R}'_j)$
- \* Use importance sampling to kill or replicate the configuration  $\Psi(\tau_{n+1}, \mathbf{R}'_j)$

- Every 20–40 time steps

- \* compute  $\Psi^\dagger(\tau_n, \mathbf{R}_j)H\Psi_V^\dagger(\tau_n, \mathbf{R}_j)$
- \* check that total number of configurations is staying reasonably constant



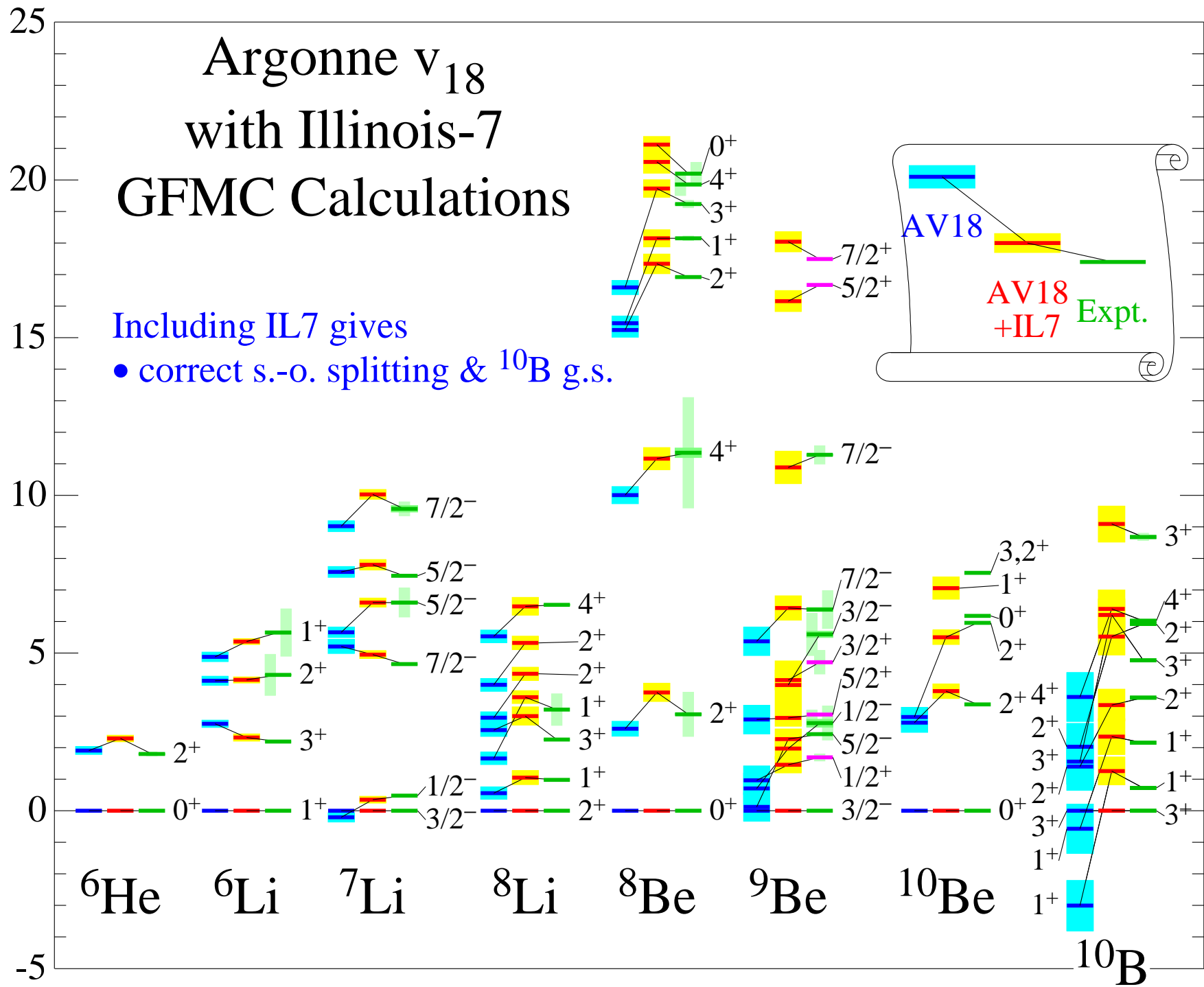


# Argonne $v_{18}$ with Illinois-7 GFMC Calculations

Excitation energy (MeV)

Including IL7 gives  
 • correct s.-o. splitting &  $^{10}\text{B}$  g.s.

AV18  
 AV18 + IL7  
 Expt.



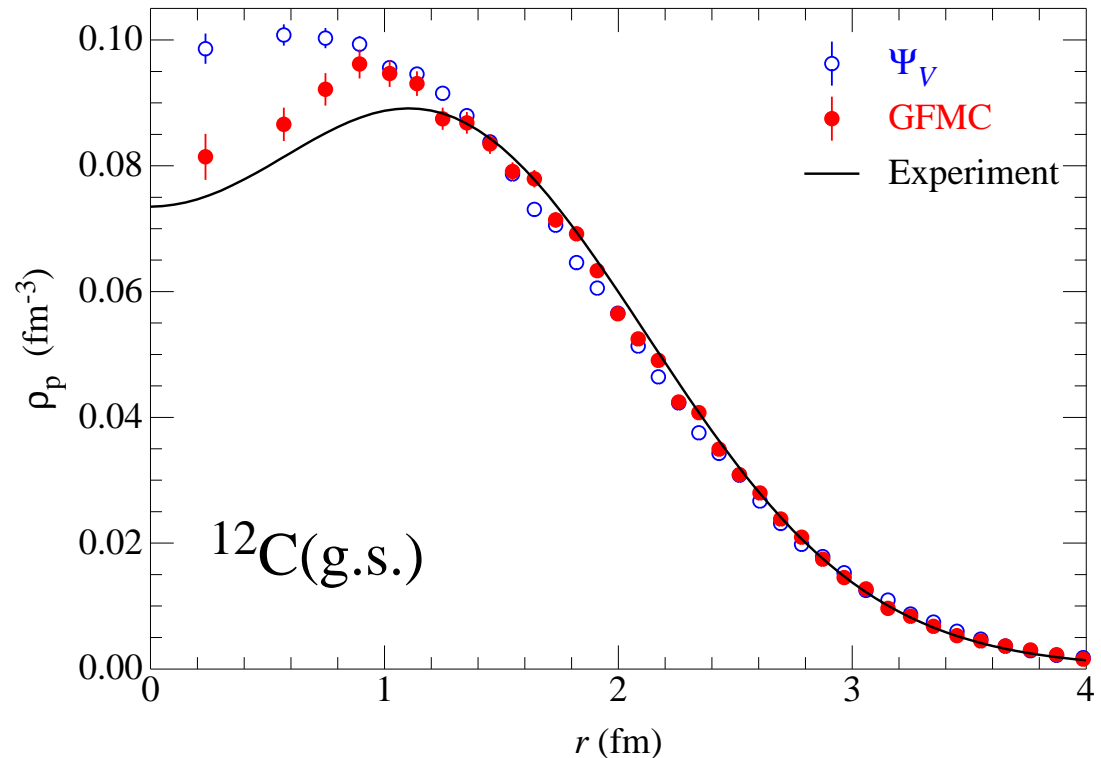
# MAKING GFMC WORK ON 131,072 PROCESSORS AND $^{12}\text{C}$

UNEDF SciDAC grant to develop  
 general-purpose load-balancing library  
 (ADLB) to run under MPI on 32,768 nodes  
 with OpenMP for 4 cores/node

INCITE grant of Argonne BG/P time used  
 for  $^{12}\text{C}$  calculations



- AV18+IL7 Hamiltonian
- $\Psi_V$  has 3- $\alpha$  structure and complete set of  $0^+$   $p$ -shell states
- Extensive convergence studies made
- GFMC generates central density dip



	VMC	GFMC	Expt.
$E$ (MeV)	-65.8(2)	-93.2(6)	-92.16
$\langle r^2 \rangle^{1/2}$ (fm)	2.36	2.35	2.33

# CHARGE SYMMETRY BREAKING

GFMC isovector and isotensor energy coefficients  $a_{A,T}^{(n)}$

$$E_{A,T}(T_z) = \sum_{n \leq 2T} a_{A,T}^{(n)} Q_n(T, T_z)$$

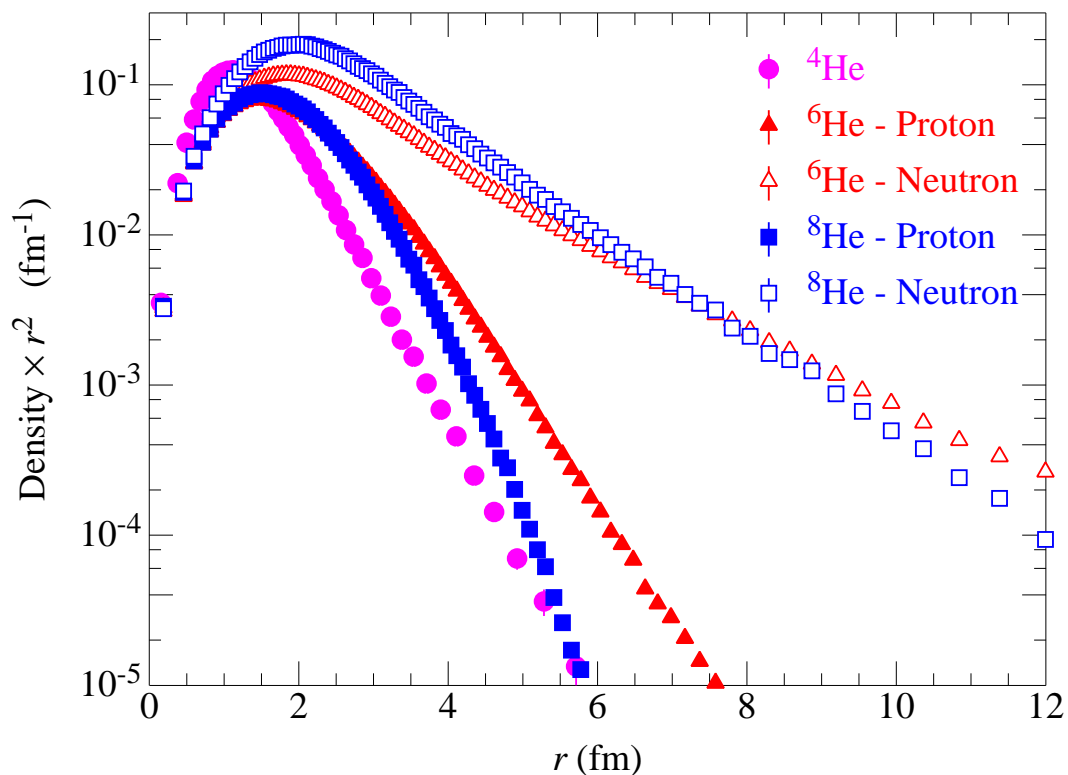
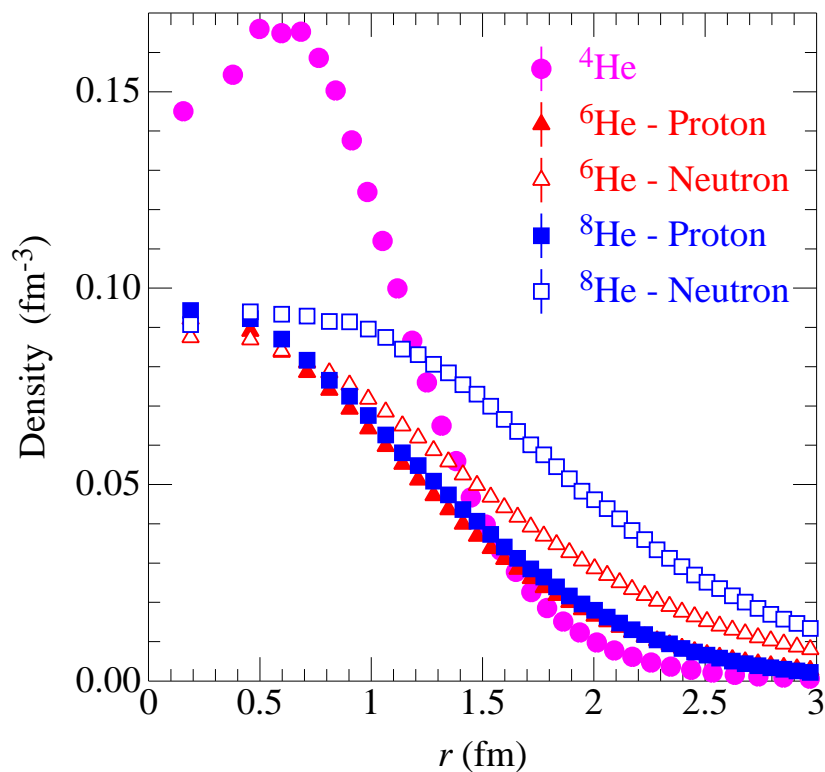
$$Q_0 = 1; \quad Q_1 = T_z; \quad \text{and} \quad Q_2 = \frac{1}{2}(3T_z^2 - T^2)$$

Argonne  $v_{18} + \text{IL7}$  (keV)

	$T$	$n$	$v^{\text{Coul}}$	$v^{\text{MM+}}$	$v^{\text{CSB+CD}}$	$K^{\text{CSB}}$	Total	Expt.
${}^3\text{H}-{}^3\text{He}$	$\frac{1}{2}$	1	650	28	65	14	757	764
${}^7\text{Li}-{}^7\text{Be}$	$\frac{1}{2}$	1	1446	36	86	23	1592	1644
${}^7\text{He}, {}^7\text{Li}, {}^7\text{Be}, {}^7\text{B}$	$\frac{3}{2}$	1	1270	9	52	17	1348	1376
	$\frac{3}{2}$	2	129	7	38		174	174
${}^8\text{Li}, {}^8\text{Be}, {}^8\text{B}$	1	1	1660	19	78	23	1780	1770
	1	2	137	4	-10		132	127
${}^8\text{He}, {}^8\text{Li}, {}^8\text{Be}, {}^8\text{B}, {}^8\text{C}$	2	1	1624	8	71	22	1726	1659
	2	2	144	6	39		189	153

# SINGLE-NUCLEON DENSITIES

$$\rho_{p,n}(r) = \sum_i \langle \Psi | \delta(r - r_i) \frac{1 \pm \tau_i}{2} | \Psi \rangle$$



RMS radii

	$r_n$	$r_p$	$r_c$	Expt
${}^4\text{He}$	1.45(1)	1.45(1)	1.67(1)	1.681(4)*
${}^6\text{He}$	2.86(6)	1.92(4)	2.06(4)	2.072(9)†
${}^8\text{He}$	2.79(3)	1.82(2)	1.94(2)	1.961(16)‡

\*Sick, PRC **77**, 041302(R) (2008)

†Wang, *et al.*, PRL **93**, 142501 (2004)

‡Mueller, *et al.*, PRL **99**, 252501 (2007)

# IS AN ALPHA PARTICLE IN A SEA OF NEUTRONS STILL AN ALPHA PARTICLE?

The  $\alpha$  core of  ${}^6,8\text{He}$  is pushed around by the neutrons

– rms charge radius changed just by C.M. effects

$\rho_{pp}(r_{12})$  is probability of two protons separated by  $r_{12}$

– not effected by C.M. motion of the  $\alpha$ .

Only pp pair in  ${}^4,6,8\text{He}$  is “in” the  $\alpha$  core

–  $\rho_{pp}(r_{12})$  measures size of  $\alpha$  core

$\rho_{pp}$  less peaked in  ${}^6,8,10\text{He}$

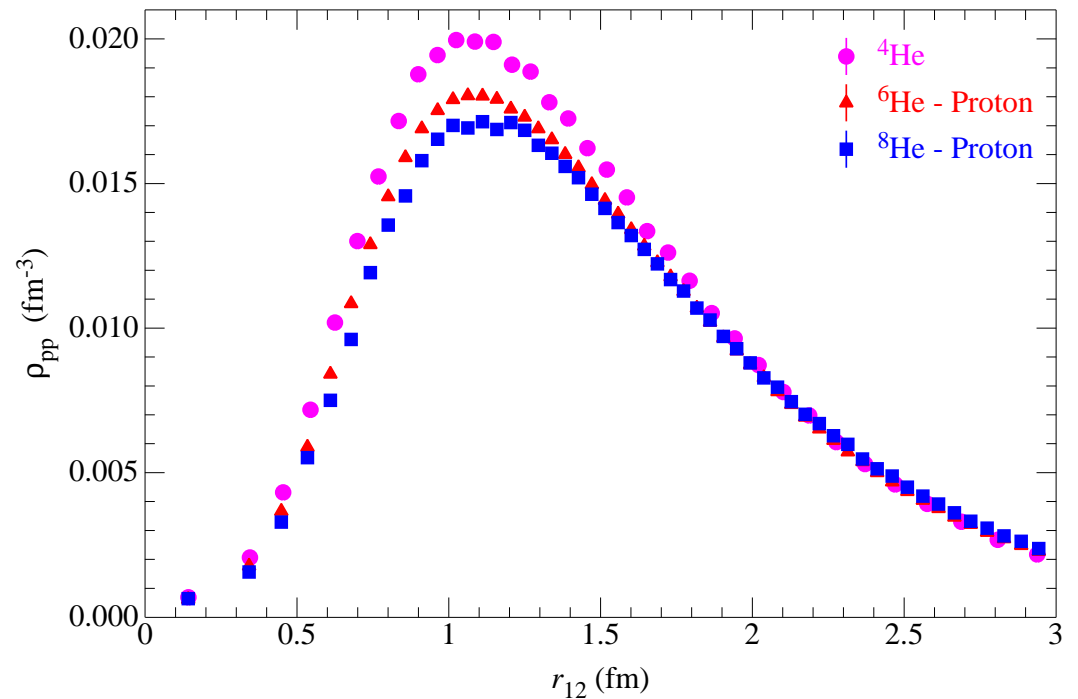
– Core polarization

– Charge-exchange correlations

with valence neutrons

– Implies 80–350 keV excitation of  $\alpha$  or

0.4–2% admixture of 20 MeV excited state



# INTRINSIC DENSITY OF ${}^8\text{Be}$

${}^8\text{Be}$  w.f.:  ${}^4\text{He}$  core + 4 p-shell nucleons + pair corr.

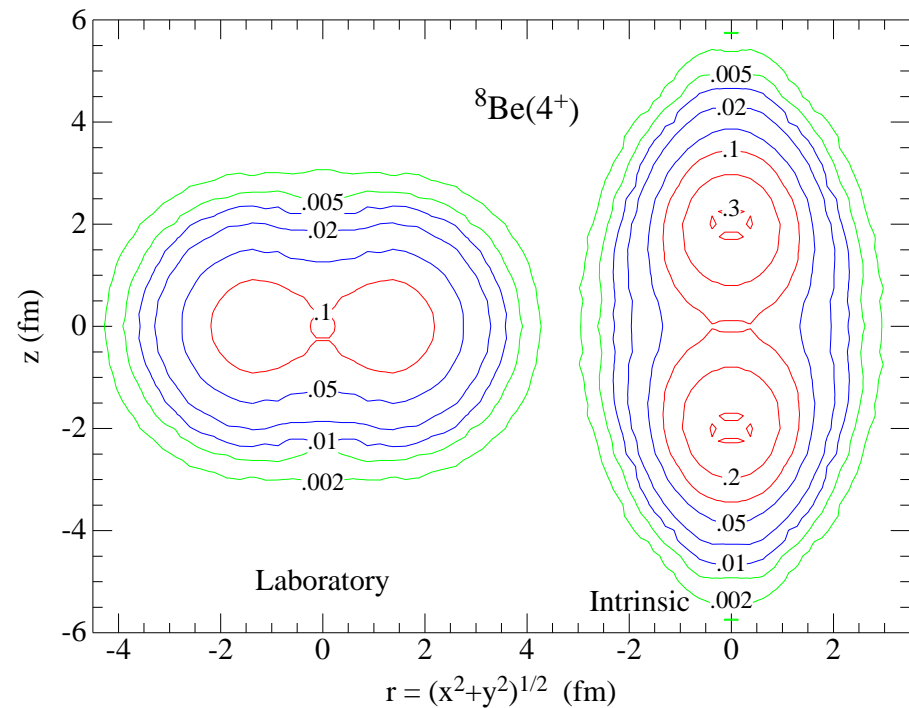
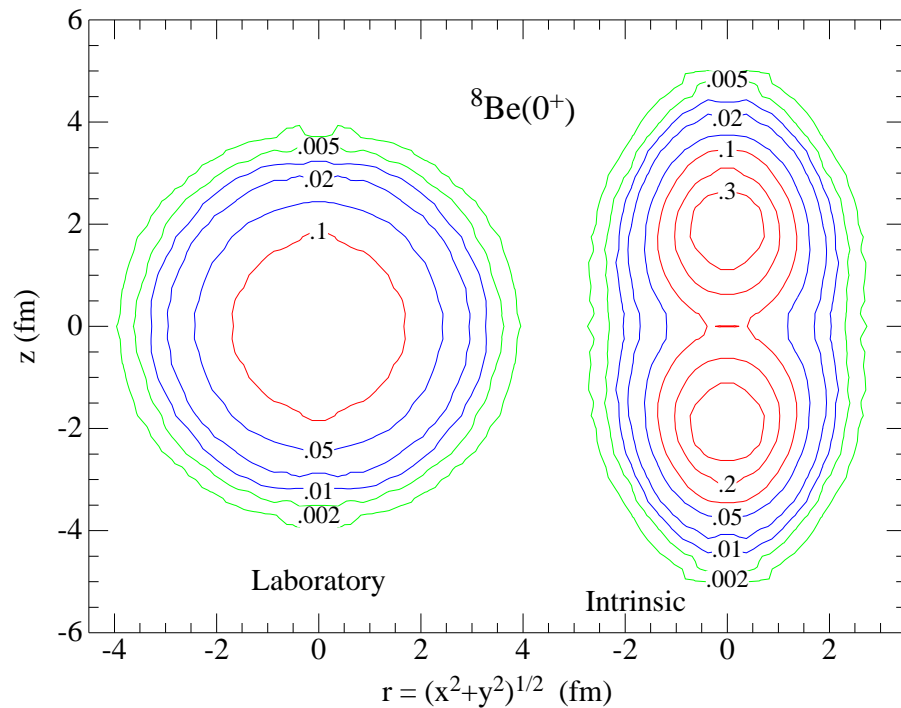
M. C.  $\rho(\mathbf{r})$ : random walk in  $|\Psi(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_8)|^2$  & periodically for each set  $(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_8)$

Lab  $\rho(\mathbf{r})$ : bin  $\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_8$

Intrinsic  $\rho(\mathbf{r})$ : find eigenvectors of moment of inertia matrix:

$$\mathcal{M} = \sum_i \begin{pmatrix} x_i^2 & x_i y_i & x_i z_i \\ y_i x_i & y_i^2 & y_i z_i \\ z_i x_i & z_i y_i & z_i^2 \end{pmatrix},$$

rotate to them, and bin  $\mathbf{r}'_1, \mathbf{r}'_2, \dots, \mathbf{r}'_8$ .



# M1, E2, F, GT transitions

NO EFFECTIVE CHARGES!

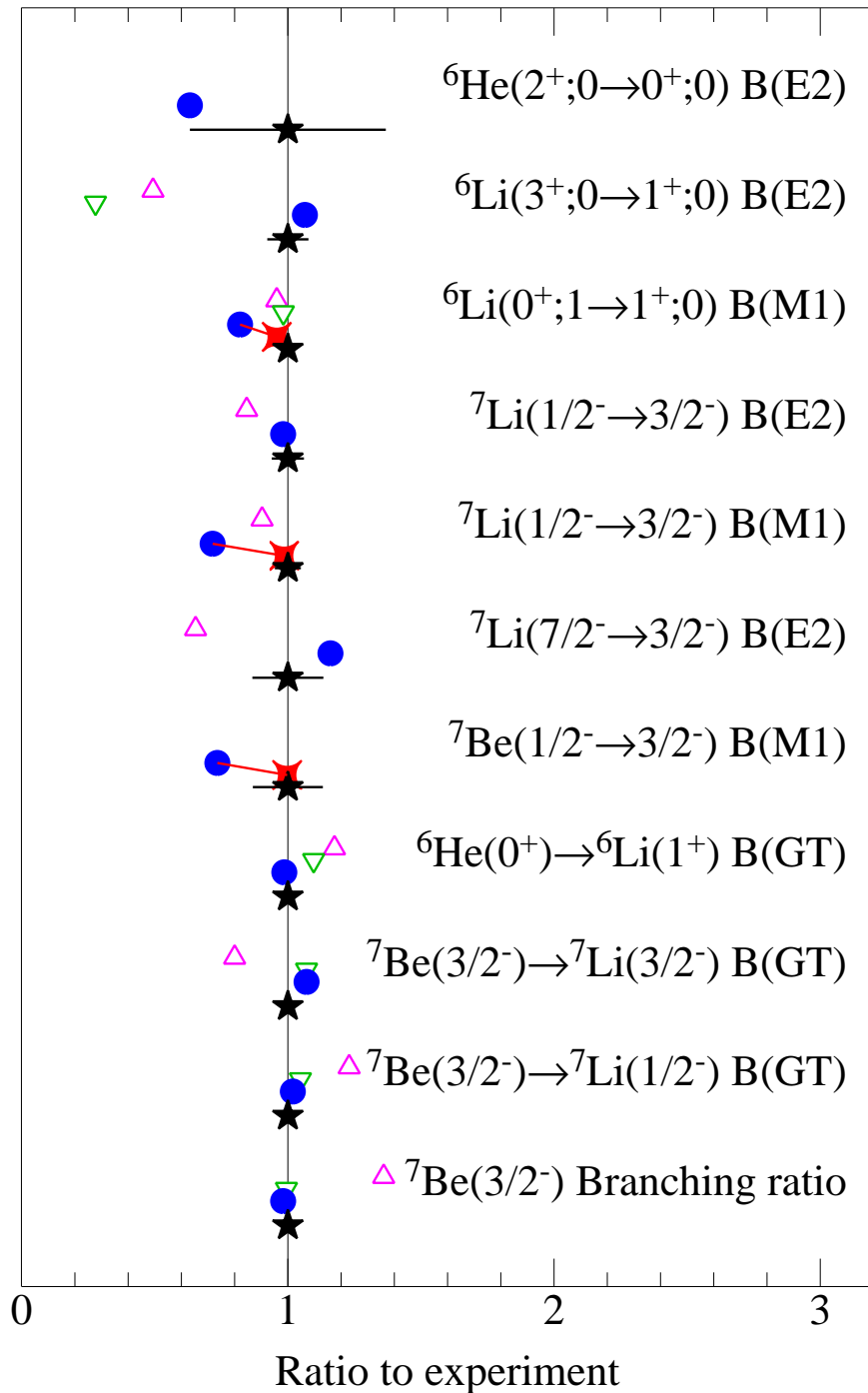
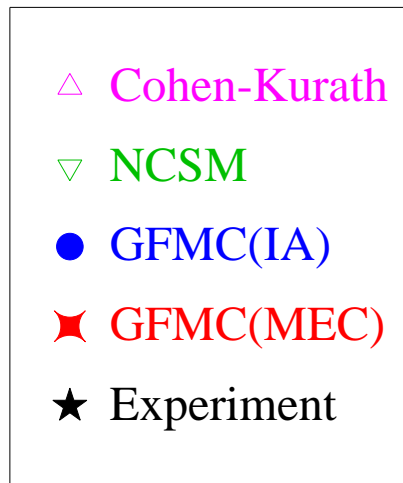
$$E2 = e \sum_k \frac{1}{2} [r_k^2 Y_2(\hat{r}_k)] (1 + \tau_{kz})$$

$$M1 = \mu_N \sum_k [ (L_k + g_p S_k)(1 + \tau_{kz})/2 + g_n S_k (1 - \tau_{kz})/2 ]$$

$$F = \sum_k \tau_{k\pm} ; \text{GT} = \sum_k \sigma_k \tau_{k\pm}$$

Pervin, Pieper & Wiringa, PRC **76**, 064319 (2007)

Marcucci, *et al.*, PRC **78**, 065501 (2008)



# RADIATIVE CAPTURE REACTIONS

$$\sigma(E_{cm}) = \frac{8\pi}{3} \frac{\alpha}{v_{rel}} \frac{q}{1 + q/m_{Li}} \sum_{LSJ\ell} \left[ \left| E_{\ell}^{LSJ}(q) \right|^2 + \left| M_{\ell}^{LSJ}(q) \right|^2 \right]$$

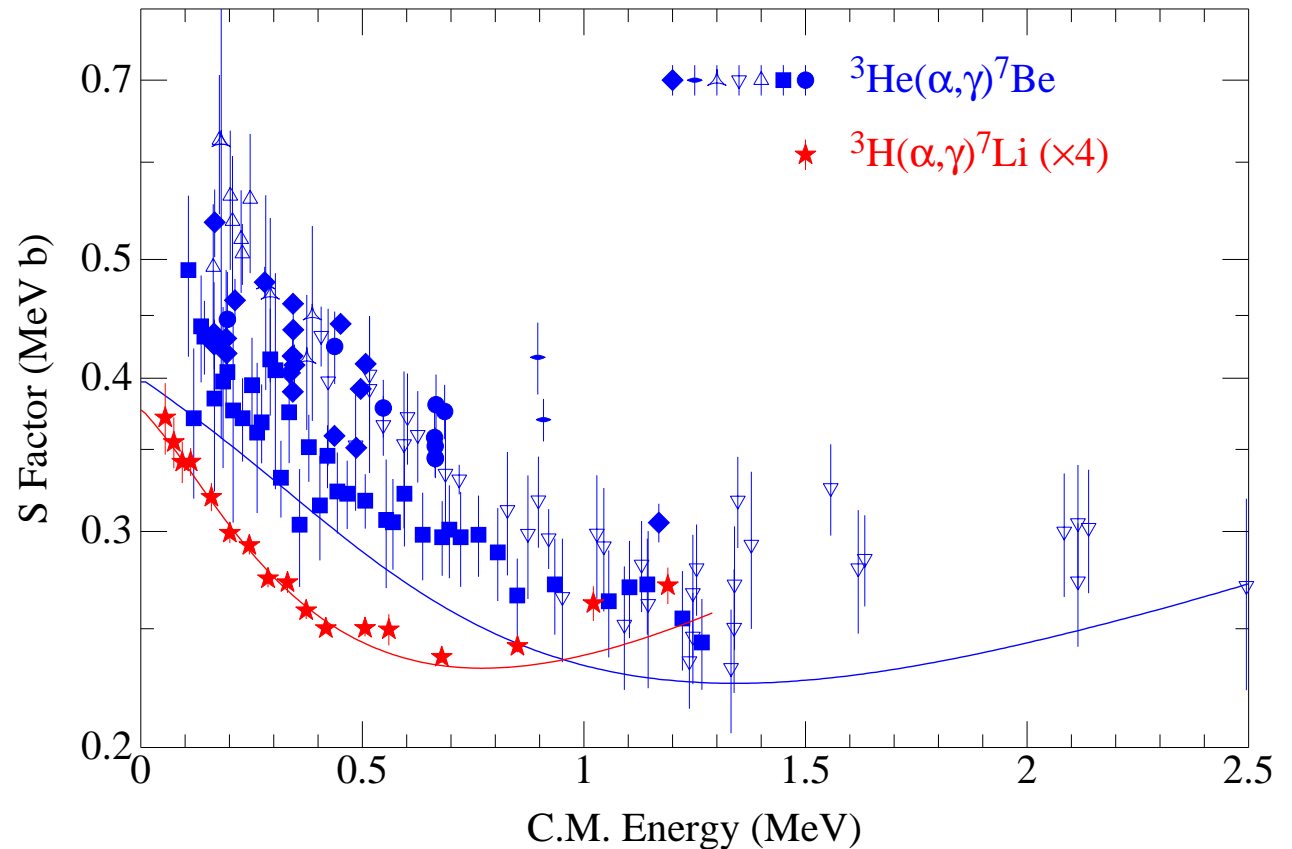
${}^3\text{H}(\alpha, \gamma){}^7\text{Li}$      ${}^3\text{He}(\alpha, \gamma){}^7\text{Be}$

Sources of  ${}^7\text{Li}$  in big bang  
(20-500 keV)

${}^7\text{Be}$  key to solar  $\nu_e$  production  
(20 keV)

Full  $A=7$  VMC calculation  
except 3+4 scattering w.f.  
from optical potential

We want to do everything including  
the 3+4 scattering in full GFMC



# GFMC FOR SCATTERING STATES

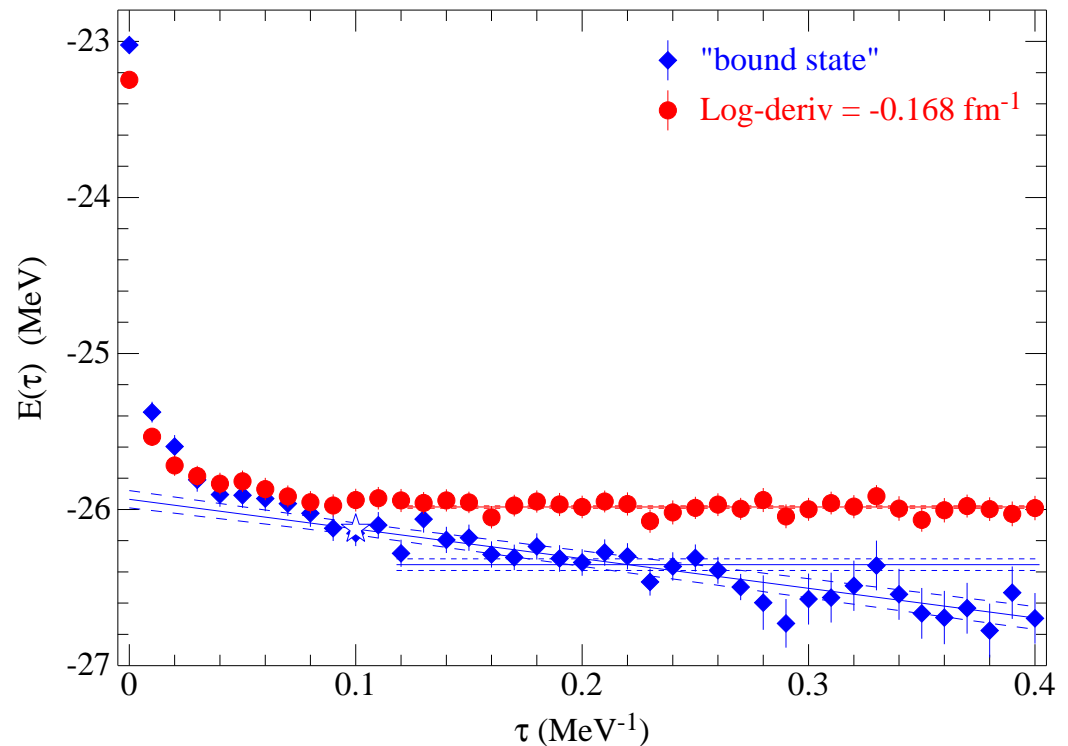
GFMC treats nuclei as particle-stable system – should be good for energies of narrow resonances  
Need better treatment for locations and widths of wide states and for capture reactions

## METHOD

- Pick a logarithmic derivative,  $\chi$ , at some large boundary radius ( $R_B \approx 9$  fm)
- GFMC propagation, using method of images to preserve  $\chi$  at  $R$ , finds  $E(R_B, \chi)$
- Phase shift,  $\delta(E)$ , is function of  $R_B, \chi, E$
- Repeat for a number of  $\chi$  until  $\delta(E)$  is mapped out
- need  $E$  accurate to  $\sim 1/3\%$

Example for  ${}^5\text{He}(\frac{1}{2}^-)$

- “Bound-state” boundary condition does not give stable energy; Decaying to  $n+{}^4\text{He}$  threshold
- Scattering boundary condition produces stable energy.



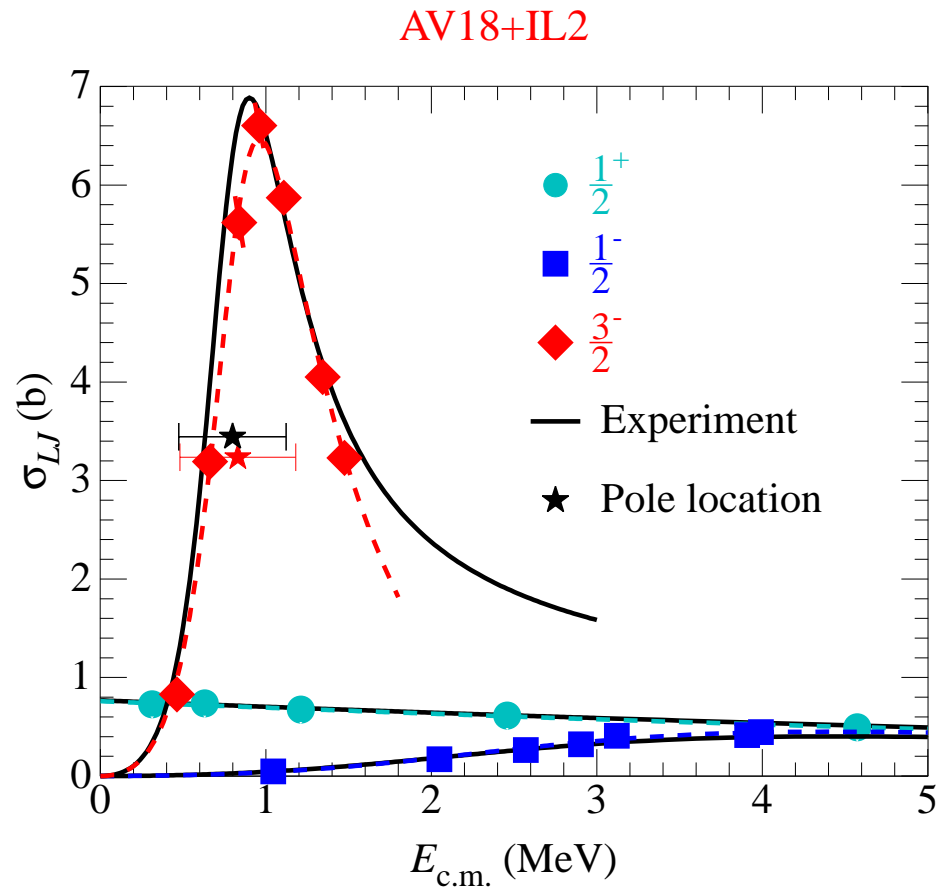
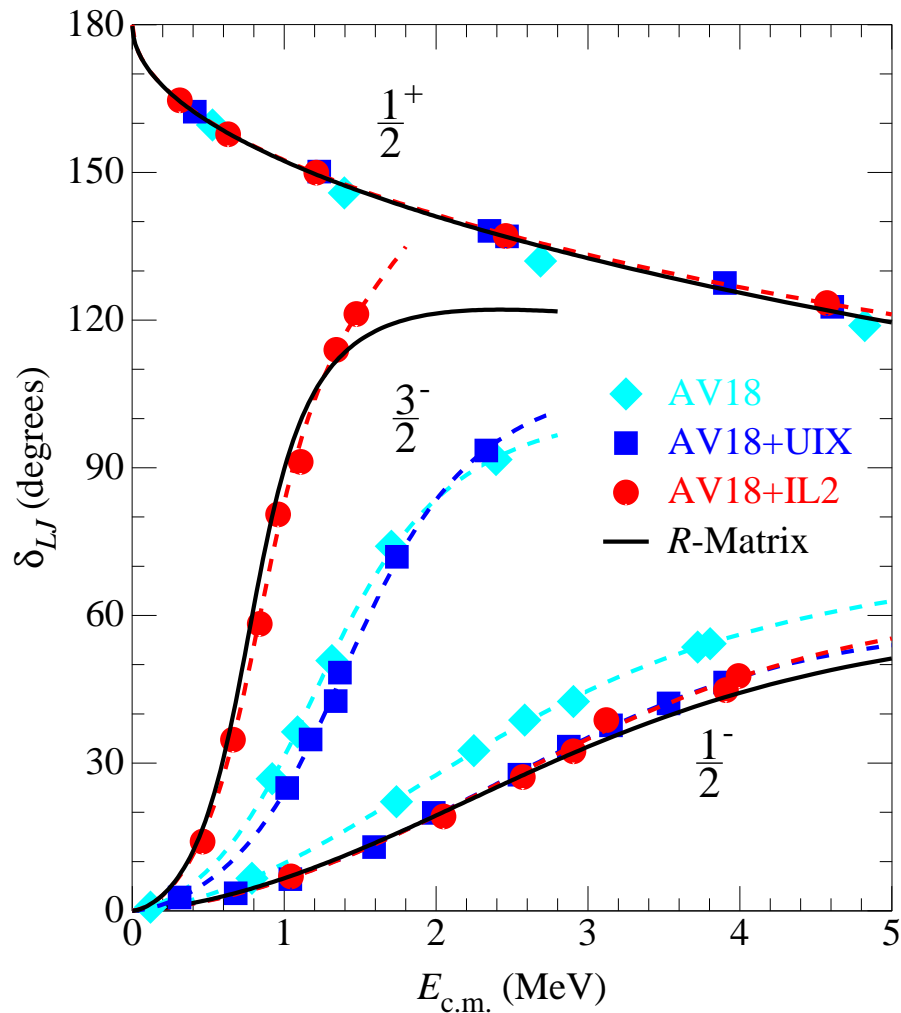
# ${}^5\text{He}$ AS $n+{}^4\text{He}$ SCATTERING

Black curves: Hale phase shifts from  $R$ -matrix analysis up to  $J = \frac{9}{2}$  of data

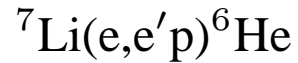
AV18 with no  $V_{ijk}$  underbinds  ${}^5\text{He}(3/2^-)$  & overbinds  ${}^5\text{He}(1/2^-)$

AV18+UIX improves  ${}^5\text{He}(1/2^-)$  but still too small spin-orbit splitting

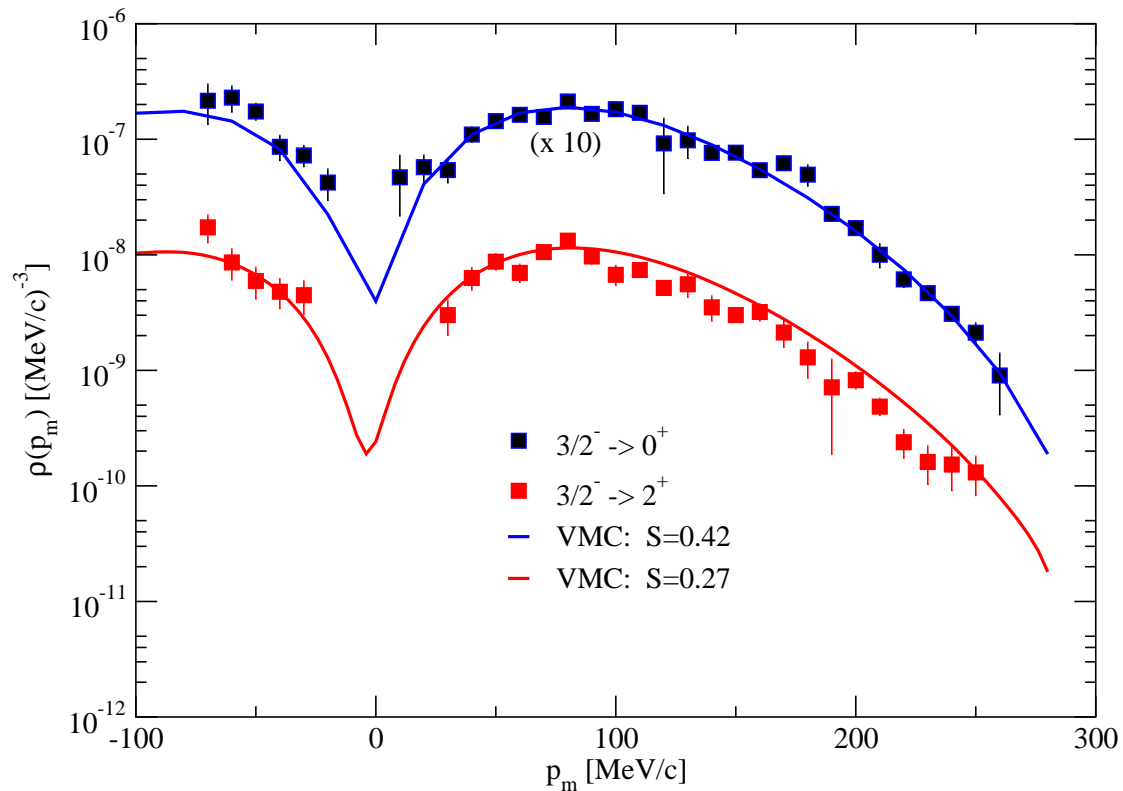
AV18+IL2 reproduces locations and widths of both  $P$ -wave resonances



# ABSOLUTE SPECTROSCOPIC FACTORS



Variational  ${}^7\text{Li}$  and  ${}^6\text{He}$  wave functions used to compute one-nucleon quasi-particle w.f.  
Normalization (Spectroscopic Factor < 50%) is a result of many-nucleon correlations.  
Standard (e,e'p) CDWIA used for cross sections.



Lapikás, Wesseling & Wiringa, Phys. Rev. Lett. **82**, 4404 (1999)

New GFMC calculations give same spectroscopic factors, better asymptotic tail

# ANALYSIS OF LIGHT-ION REACTIONS

The study of nuclei far from stability has renewed interest in  $(d,p)$  and related experiments

Calculations performed for four RIB experiments:

${}^8\text{Li}(d,p){}^9\text{Li}$  &  ${}^6\text{He}(d,p){}^7\text{He}$  &  ${}^8\text{Li}(d,{}^3\text{He}){}^7\text{He}$  (ATLAS)

${}^9\text{Li}(d,t){}^8\text{Li}$  (TRIUMF)

- PTOLEMY DWBA calculations
- $(d,p)$  vertex from AV18
- $({}^8\text{Li}, {}^9\text{Li})$ ,  $({}^7\text{He}, {}^8\text{Li})$ ,  $({}^6\text{He}, {}^7\text{He})$ ,  $(d,t)$  &  $(d,{}^3\text{He})$  vertices computed as  $A$ -body overlaps using VMC  $\langle \Psi_V(A-1) | a | \Psi_V(A) \rangle$
- Spin assignments for  ${}^9\text{Li}^*$  clarified
- Strong rejection of previously claimed  $560 \text{ keV } \frac{1}{2}^-$  state in  ${}^7\text{He}$

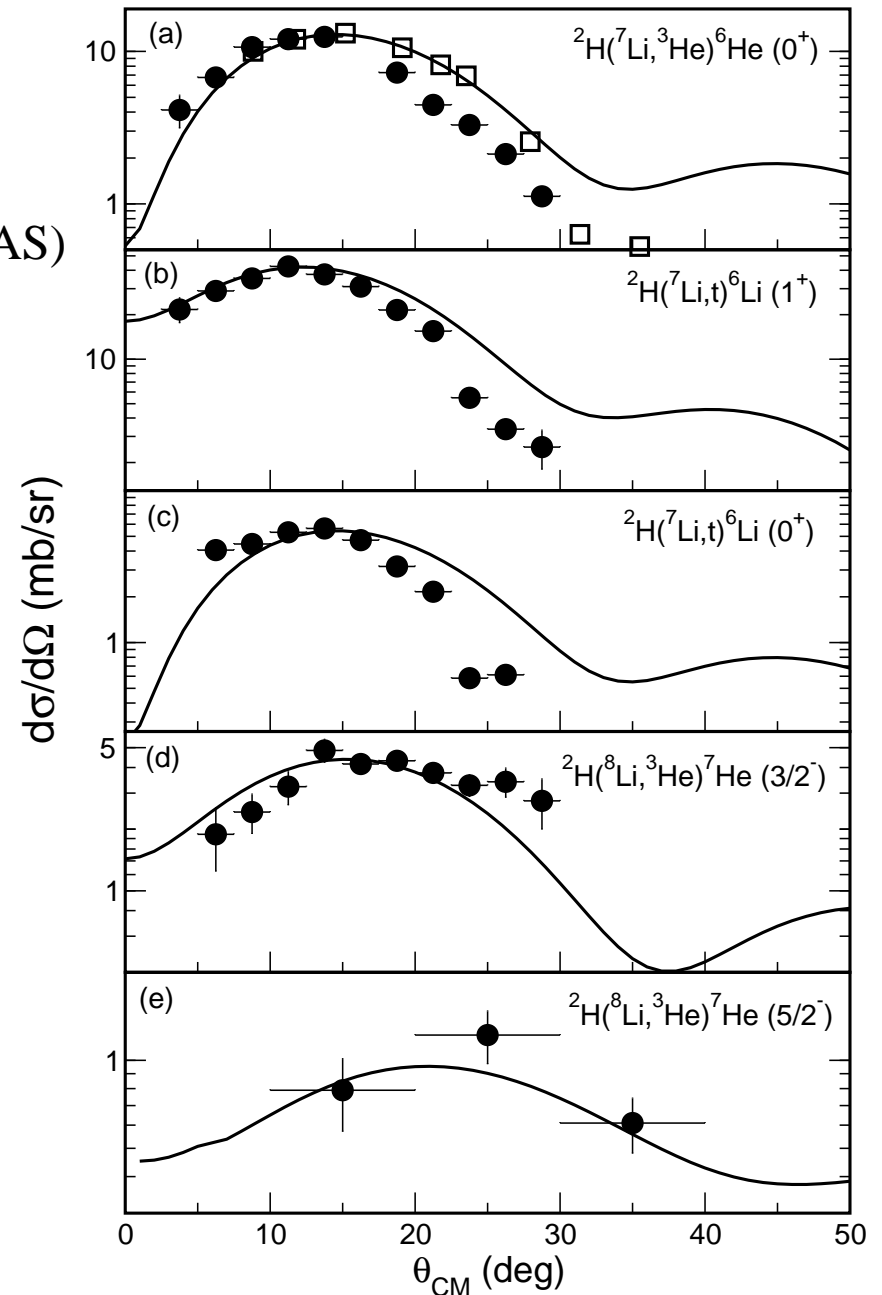
Wuosmaa *et al.*, PRL **94**, 082502 (2005)

Wuosmaa *et al.*, PRC **72**, 061301(R) (2005)

Wuosmaa *et al.*, PRC **78**, 041302(R) (2008)

Kanungo *et al.*, PLB **660**, 26 (2008)

Macfarlane & Pieper, PTOLEMY, ANL-76-11, Rev. 1 (1978)



# CAN MODERN NUCLEAR HAMILTONIANS TOLERATE A BOUND TETRANEUTRON?

AV18 + IL2 does not bind  ${}^4\text{n}$ ;  $E \sim +2$  MeV

Attempt minimal modifications to AV18+IL2 to give  $E({}^4\text{n}) \sim -0.5$  MeV

Check effects for other nuclei

Modify  ${}^1S_0$  or  ${}^3P_J$  part of AV18

- ${}^1S_0$  binds  ${}^2\text{n}$
- ${}^3P_J$  must be insanely strong

Add  $V_{ijk}(T = \frac{3}{2})$  attraction

- No effect on  $NN$  scattering
- No effect on  ${}^3\text{H}$ ,  ${}^3\text{He}$ ,  ${}^4\text{He}$
- Huge effects elsewhere

Add  $V_{ijkl}(T = 2)$  attraction (not shown)

- No effect on  ${}^4\text{H}$ ,  ${}^5\text{He}$ ,  ${}^6\text{Li}$
- Extreme (GeV scale) binding of  ${}^{5,6,8}\text{n}$ ,  ${}^6\text{He}$ , etc.

A bound  ${}^4\text{n}$  is incompatible with our understanding of nuclear forces.

Pieper, Phys. Rev. Lett. **90**, 252501 (2003)

